# Cluster Abundance and Large Scale Structure

Jiun-Huei Proty Wu

*Astronomy Department, University of California, Berkeley, CA 94720-3411, USA*

**Abstract.** We use the presently observed number density of large X-ray clusters and linear mass power spectra to constrain the shape parameter ($\Gamma$), the spectral index ($n$), the amplitude of matter density perturbations on the scale of $8h^{-1}$Mpc ($\sigma_8$), and the redshift distortion parameter ($\beta$). The non-spherical-collapse model as an improvement to the Press-Schechter formula is used and yields significantly lower $\sigma_8$ and $\beta$. An analytical formalism for the formation redshift of halos is also derived.

One of the most important constraints on models of structure formation is the observed abundance of galaxy clusters. Because they are the largest virialized objects in the universe, their abundance can be simply predicted by and thus used to constrain the linear perturbation theory. In the light of the new observations and the improvement in modeling cluster evolution, we revisit this application [1], which has been extensively explored in the literature.

Based on the maximum-likelihood analysis, we first use the observed linear mass power spectra $P(k)$ by Peacock & Dodds [2] (PD, combination of galaxy surveys) and by Hamilton, Tegmark, & Padmanabhan [3] (HTP, based on PSCz [4]; see Figure 1), to estimate the spectral index $n$, the shape parameter $\Gamma$, and the amplitude of perturbations $\sigma_8$ in the parameterization of the standard model $P(k) \propto \sigma_8^2 k^n T^2(k/\Gamma)$ [1]. The results are shown in Figure 1 and Table 1, with $\sigma_{8(\mathrm{I})} = 0.78 \pm 0.26$ for IRAS galaxies. The degeneracy between $\Gamma$ and $n$ is clear, motivating us to find the theoretically expected 'degenerated' shape parameter $\Gamma' = 0.247\Gamma\exp(1.4n) = 0.220^{+0.036}_{-0.031}$, which has a much more constrained likelihood. These results are consistent with the current constraints from CMB [5].

Following a similar formalism as in Ref. [6], we then derive the probability dis-

**TABLE 1.** Best fits of different data sets (all errors at 95% confidence level).

| | $n$ | $\Gamma$ | $\Gamma'$ | $\chi^2$/degrees of freedom (conf. level) |
|---|---|---|---|---|
| HTP | $0.91^{+1.09}_{-0.91}$ | $0.18^{+0.74}_{-0.18}$ | $0.160^{+0.085}_{-0.051}$ | 15.4/19 (70%) |
| PD | $0.99^{+0.81}_{-0.86}$ | $0.23^{+0.55}_{-0.16}$ | $0.229^{+0.042}_{-0.033}$ | 6.95/9 (64%) |
| HTP+PD | $0.84^{+0.67}_{-0.67}$ | $0.27^{+0.42}_{-0.16}$ | $0.220^{+0.036}_{-0.031}$ | 24.6/30 (74%) |

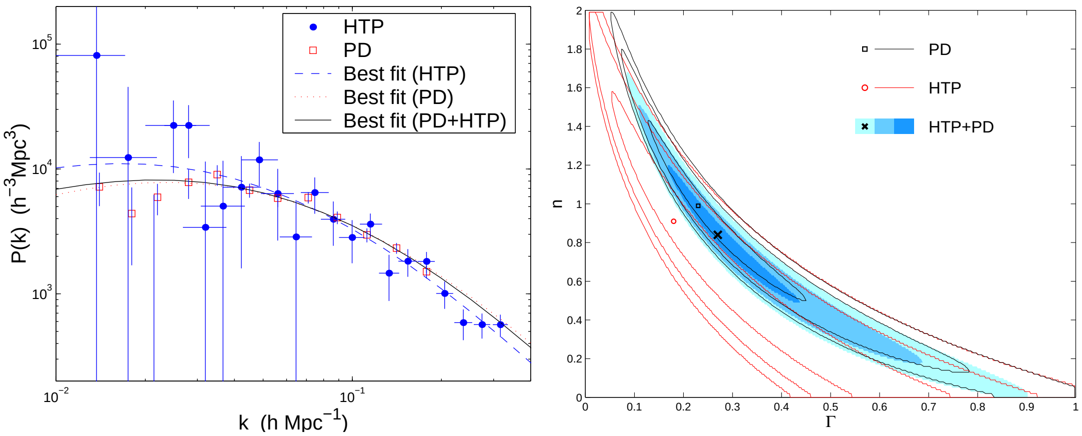


**FIGURE 1.** Matter power spectra of different observations and their best fits (left). The 68%, 95%, and 99% likelihood contours (inner out) in the $(\Gamma, n)$ parameter space (right).

**TABLE 2.** Values in the fits of $\sigma_{8(i)}$ and $\beta_{\mathrm{I}(j)}$.

| $i$ | PS | ST | LS | ST+LS | $j$ | PS | ST+LS |
|---|---|---|---|---|---|---|---|
| $c_1$ | 0.54 | 0.50 | 0.455 | 0.477 | $d_1$ | 0.693 | 0.613 |
| $c_2$ | 0.45 | 0.37 | 0.31 | 0.34 | $d_2$ | 0.26 | 0.24 |

tribution function $p_{z(i)}(z)$ of cluster formation redshift $z$ [1] for different models of mass function $n_i(M)$, where $i$ =PS (Press & Schechter [7]), ST (Sheth & Tormen [8]), or LS (Lee & Shandarin [9]), the last two of which incorporate non-spherical collapse . With specified $n_i(M)$, $p_{z(i)}(z)$, $\sigma_8$, and the previously estimated $n$ and $\Gamma$ (or $\Gamma'$), we can project the present cluster abundance of a given mass $M$ into the space of formation redshift $z$, and then use the virial mass-temperature relation to associate this abundance with the virial temperature $T$ that corresponds to the given $M$ and $z$. An integration over $T$ and $z$ will give us a prediction of cluster abundance, which is a function of $\sigma_8$. A comparison of this with the observation [10] will give the normalization of $\sigma_8$. Combined with the $\sigma_{8(\mathrm{I})}$ estimated earlier, it further yields the constraint on the redshift distortion parameter $\beta_{\mathrm{I}} \approx \Omega_{\mathrm{m}}^{0.6}\sigma_8/\sigma_{8(\mathrm{I})}$, which quantifies the confusion between the Hubble expansion and the local gravitational collapse [11]. Our results can be fitted by $\sigma_{8(i)}(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}) = c_1 \Omega_{\mathrm{m0}}^{\alpha}$, where $i$ =PS, ST, LS or ST+LS, and $\alpha \equiv \alpha(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}) = -0.3 - 0.17\Omega_{\mathrm{m0}}^{c_2} - 0.13\Omega_{\Lambda 0}$ (see Figure 2 left), and $\beta_{\mathrm{I}(j)}(\Omega_{\mathrm{m0}}, \Omega_{\Lambda 0}) = d_1 \Omega_{\mathrm{m0}}^{d_2 - 0.16(\Omega_{\mathrm{m0}} + \Omega_{\Lambda 0})}$, where $j$ =PS or ST+LS (see Figure 2 right). The parameter values of these fits are given in table 2.

It is clear that the $\sigma_8$ and $\beta_{\mathrm{I}}$ resulted from non-spherical-collapse models (ST and LS) are systematically lower than those based on the PS formalism, mainly owing to the larger mass function on cluster scales. A detailed investigation of the uncertainties in our final results shows that the main contributor is the uncertainty in the normalization of the virial mass-temperature relation. Therefore further improvement in this normalization will provide us with more stringent constraint on both $\sigma_8$ and $\beta_{\mathrm{I}}$. In addition, since we saw significant corrections in the resulting

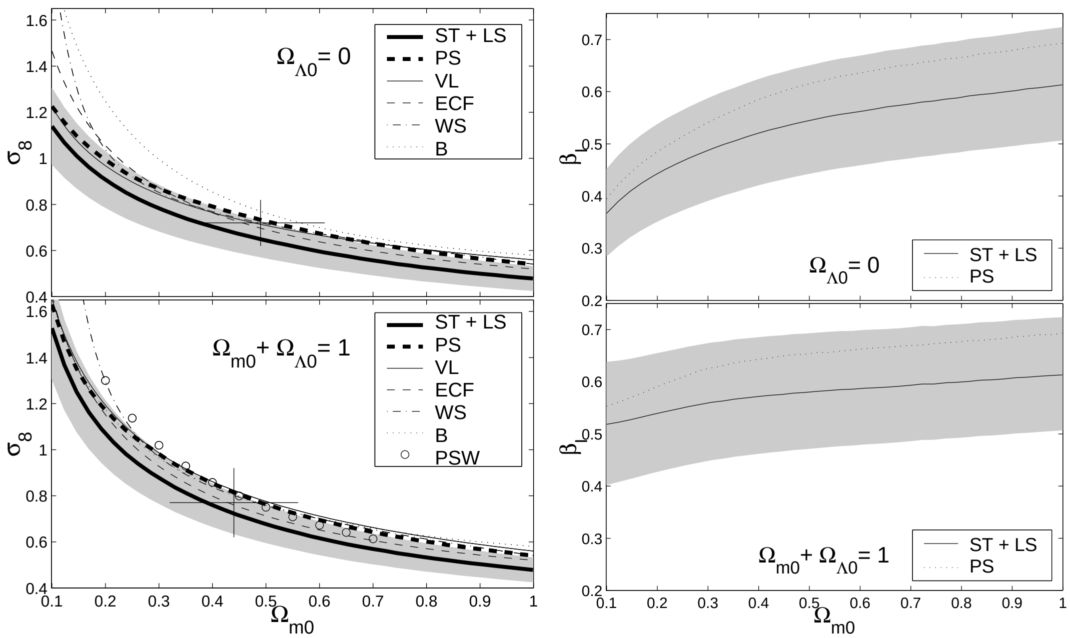


**FIGURE 2.** The cluster-abundance-normalized $\sigma_8$ and $\beta_\mathrm{I}$, in comparison with results of $\sigma_8$ from the literature (see [12] for the abbreviations used in the figure legend).

$\sigma_8$ and $\beta_\mathrm{I}$ when switching from the PS formalism to the more accurate non-spherical-collapse models, we urge the use of these models in all relevant studies, especially when we are entering the regime of precision cosmology. We acknowledge the support from NSF KDI Grant (9872979) and NASA LTSA Grant (NAG5-6552).

# REFERENCES

1. Wu, J. H. P., *astro-ph/0012207* (2000).
2. Peacock, J. A., Dodds, S. J., *MNRAS.*, **267**, 1020 (1994).
3. Hamilton, A. J. S., Tegmark, M., Padmanabhan, N., *MNRAS.*, **317**, L23 (2000).
4. Saunders, W. et al., astro-ph/0001117 (2000).
5. Jaffe, A. H. et al., *Phys.Rev.Lett.* in press, *astro-ph/0007333* (2001).
6. Lacey, C., Cole, S., *MNRAS.*, **262**, 627 (1993); *MNRAS.*, **271**, 676 (1994).
7. Press, W. H., Schechter, P., *ApJ.*, **187**, 452 (1974).
8. Sheth, R. K., Tormen, G., *MNRAS.*, **308**, 119 (1999).
9. Lee, J., Shandarin, S. F., *ApJ.*, **500**, 14 (1999).
10. Henry, J. P., *ApJ.*, **534**, 565 (2000).
11. Kaiser, N., *MNRAS.*, **227**, 1 (1987).
12. Viana, P.T.P., Liddle, A.R., *MNRAS.*, **303**, 535 (1999; VL); Eke, V.R., Cole, S., Frenk, C.S., *MNRAS.*, **282**, 263 (1996; ECF); Wang, L., Steinhardt, P.J., *ApJ.*, **508**, 483 (1998; WS); Borgani, S., Rosati, P., Tozzi, P., Norman, C., *ApJ.*, **517**, 40 (1999; B); Pierpaoli, E., Scott, D., White, M., *astro-ph/0010039* (2000; PSW).